

**MODELING TRANSONIC AERODYNAMIC RESPONSE USING  
NONLINEAR SYSTEMS THEORY FOR USE WITH  
MODERN CONTROL THEORY**

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**Title Chart**

This presentation addresses the application of a nonlinear systems theory to the modeling of nonlinear unsteady aerodynamic responses. In particular, transonic aerodynamic responses, such as those computed using CFD codes, will be modeled.

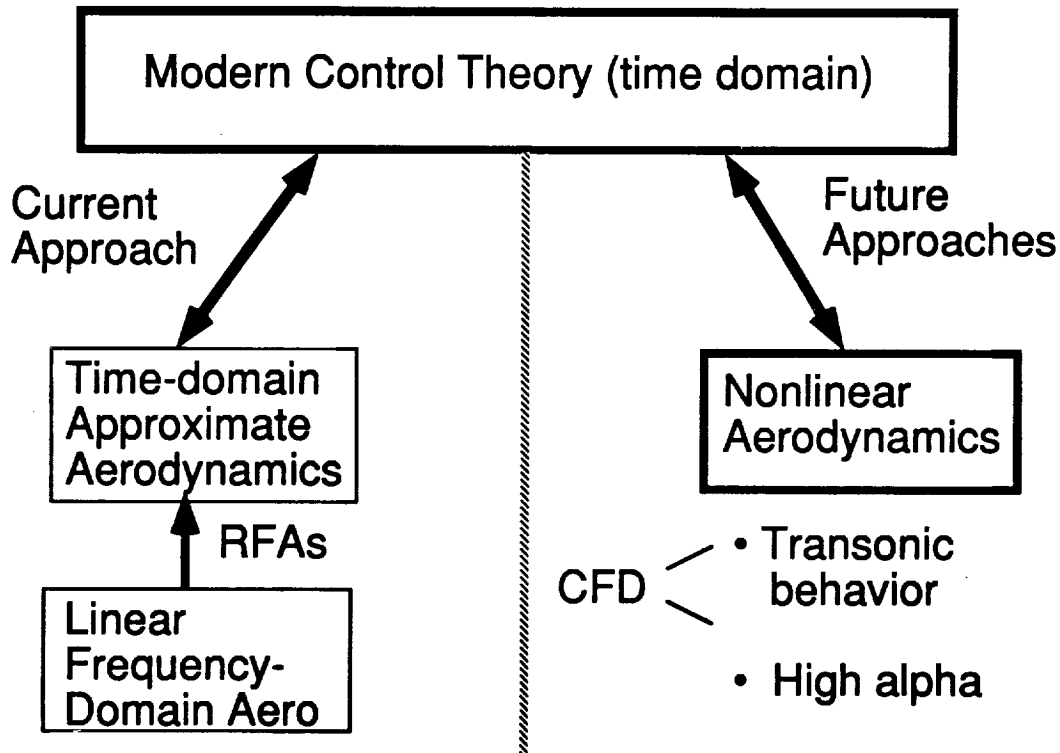
# OUTLINE

- Motivation and Approach
- Volterra Theory of Nonlinear Systems
- CAP-TSD Code
- Application to a CFD Model  
(NACA 0012 rectangular wing)
- Concluding Remarks

## Outline Chart

The presentation begins with a brief description of the motivation and approach that has been taken for this research. This will be followed by a description of the Volterra Theory of Nonlinear Systems and the CAP-TSD code which is an aeroelastic, transonic CFD (Computational Fluid Dynamics) code. The application of the Volterra theory to a CFD model and, more specifically, to a CAP-TSD model of a rectangular wing with a NACA 0012 airfoil section will be presented. Finally, some concluding remarks will be made.

## MOTIVATION FOR RESEARCH

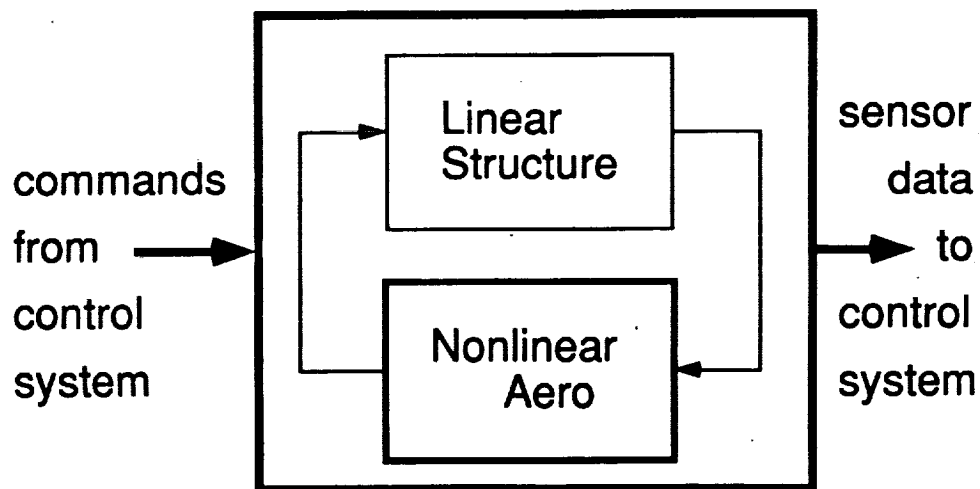


### Motivation For Research

The current approach for performing aeroservoelastic analysis and design, in the preliminary design stage, begins with the generation of linear, frequency-domain aerodynamics such as those obtained using doublet lattice theory. Using the concept of rational function approximations, a time-domain model of the linear aerodynamics is generated which is then amenable for use with modern control theory. In the future, however, it is highly desirable to be able to design control laws that can account for nonlinearities in the flow such as the nonlinearities created by transonic flows and high alpha motions. Many of these complex behaviors are currently modeled using CFD codes but there is, currently, no practical method for using the information generated by CFD codes in modern control theory.

## BASIC APPROACH

- To model unsteady nonlinear aerodynamic responses as a Volterra nonlinear system



### Basic Approach

An approach that addresses the problem mentioned in the previous chart is to model the unsteady nonlinear aerodynamic system as a Volterra nonlinear system. This system can then be coupled with a structure, usually a linear structure but this is not a hard requirement for the methodology. This aeroelastic system can then be treated as the plant for which control laws can be designed and/or evaluated. But what exactly is a Volterra nonlinear system ?

## VOLTERRA THEORY OF NONLINEAR SYSTEMS

### Volterra Series

$$\begin{aligned}
 y(t) = & \int_0^t h_1(t-\tau) u(\tau) d\tau + \\
 & \int_0^t \int_0^t h_{2s}(t-\tau_1, t-\tau_2) u(\tau_1) u(\tau_2) d\tau_1 d\tau_2 + \dots + \\
 & \int_0^t \dots \int_0^t h_{ns}(t-\tau_1, \dots, t-\tau_n) u(\tau_1) \dots u(\tau_n) d\tau_1 \dots d\tau_n + \dots
 \end{aligned}$$

- Assumes system is causal and time invariant
- Symmetric higher-order kernels:  $h_{2s}(t_1, t_2) = h_{2s}(t_2, t_1)$
- Higher-order kernels are measure of nonlinearity
- Theory also referred to as the Volterra-Wiener Theory

#### Volterra Theory of Nonlinear Systems, Volterra Series

The basic premise of the Volterra theory of nonlinear systems is that the response of a nonlinear system,  $y(t)$ , due to an arbitrary input,  $u(t)$ , can be predicted by an infinite series of multidimensional convolution integrals. This is known as the Volterra series. Each convolution integral has a kernel associated with that particular order. That is, the first integral, also referred to as the first-order integral, has the standard one-dimensional kernel or unit impulse response. The second integral, or the second-order convolution, has the second-order kernel which is a two-dimensional unit impulse response, and so on. This particular formulation assumes that the system is causal and time invariant. The higher-order kernels, of order two and above, are symmetric. These kernels are also a measure of nonlinearity. This can be clearly seen when the higher-order kernels are zero and the response of the system is linear. Therefore, when the higher-order kernels are non-zero valued, they represent a deviation from linear response or a nonlinear response. Due to the contributions of Norbert Wiener, the theory is also referred to as the Volterra-Wiener theory of nonlinear systems.

# VOLTERRA THEORY OF NONLINEAR SYSTEMS

## Volterra Series

For a "weakly" nonlinear system,

$$y(t) \cong \int_0^t h_1(t-\tau) u(\tau) d\tau + \iint_{-\infty}^t h_{2s}(t-\tau_1, t-\tau_2) u(\tau_1) u(\tau_2) d\tau_1 d\tau_2$$

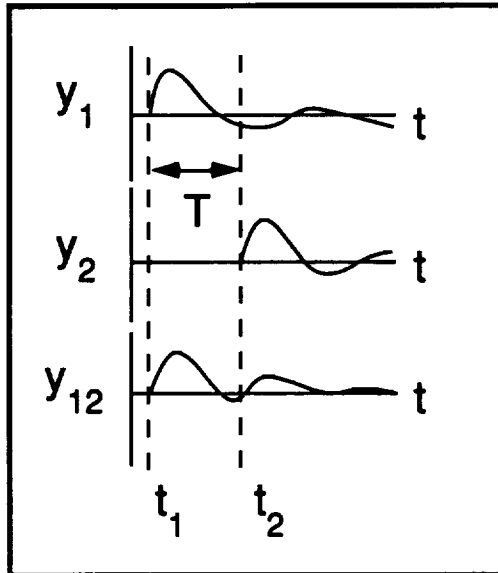
- Many physical systems accurately modeled as weakly nonlinear
- The basic problem is one of kernel identification

### Weakly Nonlinear Systems

The assumption of a weakly nonlinear system can be made in order to simplify the present analysis. This assumption simply states that kernels of order three and above are negligible and the response of the system can be modeled using only second-order nonlinearities. There exist many physical systems that have been accurately modeled as weakly nonlinear in the fields of biology, chemistry, and robotics. The basic problem, then, is one of kernel identification. If the first- and second-order kernels can be identified, then the response of the nonlinear system to arbitrary inputs can be computed.

# VOLTERRA THEORY OF NONLINEAR SYSTEMS

## Kernel Definition and Identification



$$h_{2s} = (1/2) (y_{12} - y_2 - y_1)$$

$$h_{2s} = f(t, T)$$

- For a linear system, second- (and higher-) order kernels are identically zero
- Nature of nonlinear kernels depends on the nature of the system being investigated

### Kernel Definition and Identification

One method for identifying kernels is the method of unit impulse responses. Shown in this chart is the definition of the second-order kernel, for a weakly nonlinear system, using unit impulse responses. The  $y_1$  response is the response of the nonlinear system to a unit impulse input at time  $t_1$ ;  $y_2$  is the response of the nonlinear system to a unit impulse input at time  $t_2$ ; and  $y_{12}$  is the response of the nonlinear system to a unit impulse input at time  $t_1$  and a unit impulse input at time  $t_2$ . Since the system is time invariant,  $y_2$  is  $y_1$  shifted in time. The second-order kernel is then computed as one-half the difference of these responses. As the time lag,  $T$ , between the two unit impulse inputs is varied, additional terms of the second-order kernel are generated. As can be seen, the second-order kernel is a two-dimensional function of time,  $t$ , and time lag,  $T$ . It is clear from this definition that for a purely linear system, the second-order kernel is identically zero by the principle of superposition. When this second-order kernel is non-zero, this implies a deviation from linearity, or a nonlinear response. The nature, or character, of the nonlinear kernels depends on the system being investigated and no assumptions can be made a priori to the actual computation of the kernel.

# VOLTERRA THEORY OF NONLINEAR SYSTEMS

## State-Space Realization

- Linear

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

$$\text{and } h(t) = C [ \exp(At) ] B$$

- Nonlinear (Bilinear State Equation)

$$\begin{aligned}\dot{x} &= Ax + Nxu + Bu \\ y &= Cx\end{aligned}$$

$$\text{and } h(t_1, t_2) = C[\exp(At_2)(N)\exp(At_1)]B$$

- If kernels are known, then A, B, C, and N matrices can be computed.

### State-Space Realization

A truly powerful characteristic of the Volterra theory of nonlinear systems is shown in this chart. It is well known that for a linear system described as shown here that the unit impulse response of that system is defined as shown. If the unit impulse response of the system is known, then using realization techniques one can compute the A, B, and C matrices. The analogous situation exists for a Volterra nonlinear system where the second-order kernel is defined as shown. Therefore, if the second-order kernel of a system can be identified, the A, B, C, and N matrices of a bilinear state-space equation can be realized. This is then a nonlinear, state-space description of the nonlinear unsteady aerodynamic system.



## **CAP-TSD CODE**

- **Computational Aeroelasticity Program - Transonic Small Disturbance**
- **Uses time-accurate, approximate factorization finite-difference algorithm**
- **Applicable to realistic configurations**

### **CAP-TSD Code**

The CFD code used for this research is the CAP-TSD code. CAP-TSD is an acronym that stands for Computational Aeroelasticity Program - Transonic Small Disturbance. The code solves the nonlinear, general-frequency transonic small disturbance equation using a time-accurate, approximate factorization algorithm developed by Dr. Jack Batina and a team from the Unsteady Aerodynamics Branch. The code is applicable to realistic configurations.

## APPLICATION TO A CFD MODEL

- Theory -- Continuous systems -- unit impulse function  
CFD codes -- Discrete systems -- unit pulse function  
(example in paper and Ref. 23)

$$\begin{aligned} \text{unit pulse function } u(t) &= 1.0 \text{ for } t = t_0 \\ &0.0 \text{ for } t \neq t_0 \end{aligned}$$

- Unsteady Aerodynamic System  
input -- downwash function  
output -- lift or moment response

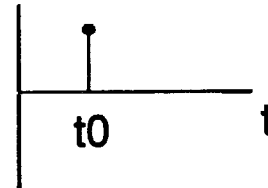
### Application to a CFD Model

The Volterra theory discussed thus far addresses continuous systems for which the unit impulse function is defined. CFD codes, however, are discrete systems. Therefore, the unit pulse function, which is the discrete equivalent of the unit impulse input for continuous systems, should be used. The unit pulse function is defined as having a value of unity at one point in time and being zero at all other times. The unsteady aerodynamic system is defined as having the downwash function as the input and lift, moment, or any other force as its output. Definition of the input and output depends on the system to be investigated.

## APPLICATION TO CAP-TSD

- Downwash function in CAP-TSD for any modeshape

$$f(x,y,t)^{\pm} = \frac{dz^{\pm}}{dx} + A_1(t) \frac{d\phi(x,y)}{dx} + A_2(t) \left( \frac{1}{L_{ref}} \right) \phi(x,y)$$

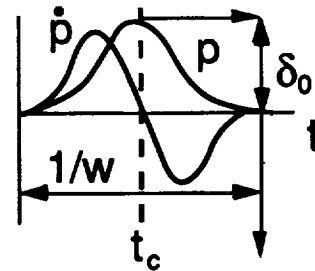


Apply unit pulse to  $A_1(t)$  and  $A_2(t)$

- Exponential pulse capability (NOT unit pulse)

$$p(t) = \delta_0 \exp(-w(t-t_c)^2)$$

$$\dot{p}(t) = -2w(t-t_c)p$$



For arbitrary pitching motion,

$$A_1(t) = p(t) \quad \text{and} \quad A_2(t) = \dot{p}(t)$$

### Application to CAP-TSD

More specifically, the application of the Volterra theory to the CAP-TSD code is shown in this chart. The downwash function is defined as shown where the plus and minus signs represent the upper and lower surfaces of the airfoil. The  $dz/dx$  term are the slopes of the upper and lower surfaces of the airfoil. The  $A_1$  term represents the rate of change of motion since it is multiplied by the modal slopes and the  $A_2$  term represents the actual motion. A unit pulse is applied to  $A_1$  and  $A_2$  separately to obtain the unit pulse response due to each of these terms of the downwash. The CAP-TSD code has a capability referred to as the exponential pulse capability which should not be confused with the unit pulse input. The exponential pulse capability is defined as shown and for arbitrary motions, the  $A_1$  term is replaced with the  $p(t)$  function and the  $A_2$  term is replaced with the rate-of-change of  $p(t)$  function.

## **RESULTS FOR NACA0012 RECTANGULAR WING Computational Model**

- NACA0012 rectangular wing with pitch and plunge degrees of freedom
- Semi-span model (panel AR=2.0)
- Grid dimensions: 140 x 40 x 92

### **Computational Model**

The CAP-TSD model consists of a rectangular wing with a NACA 0012 airfoil and with pitch and plunge degrees of freedom. It is a semi-span model with a panel aspect ratio of 2.0. The grid is dimensioned 140 by 40 by 92 grid points in the x-, y-, and z-directions respectively.

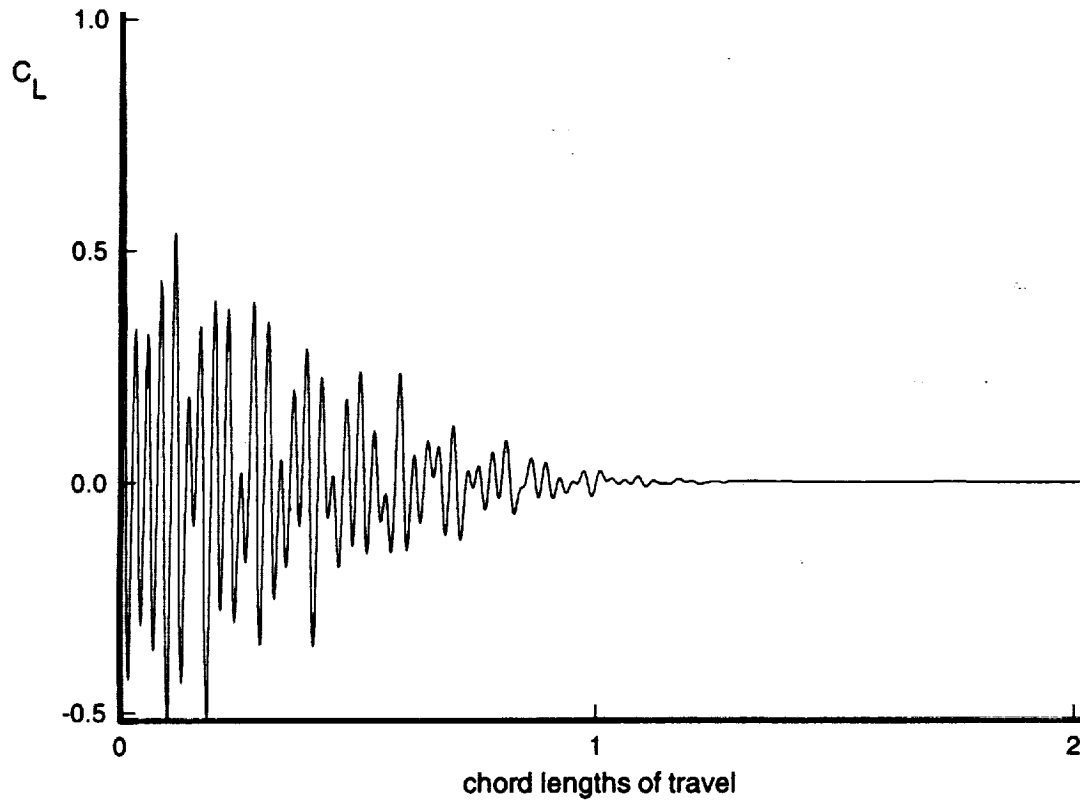
## RESULTS FOR NACA0012 RECTANGULAR WING Analysis

- Lift-coefficient response due to pitch about the mid-chord
- All responses at  $M = 0.8$
- Nonlinear responses about a converged steady-state solution

### Analysis

The results that will be presented consist of lift coefficient due to a pitching motion about the mid-chord of the wing. All results are for a Mach number of 0.8, for which a shock exists so that differences between the linear (flat plate) and nonlinear (thickness) solutions should be noticeable. All nonlinear CAP-TSD solutions were computed about a converged steady-state solution.

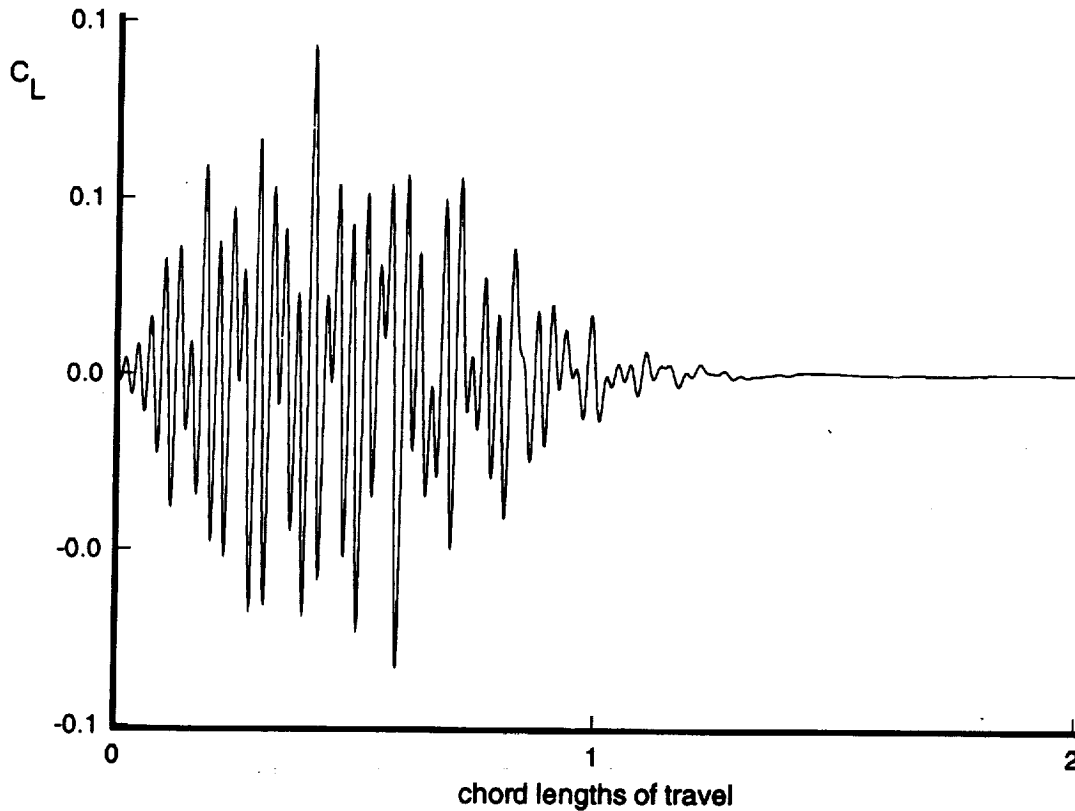
# LINEAR (FLAT PLATE) UNIT PULSE RESPONSE IN LIFT DUE TO FIRST COMPONENT OF PITCHING MOTION



Linear (flat plate) Unit Pulse Response in Lift Due to First Component of Pitching Motion

This is the unit pulse response in lift due to the first component of the pitching motion, or the downwash. The response is stable, or square integrable, as would be expected.

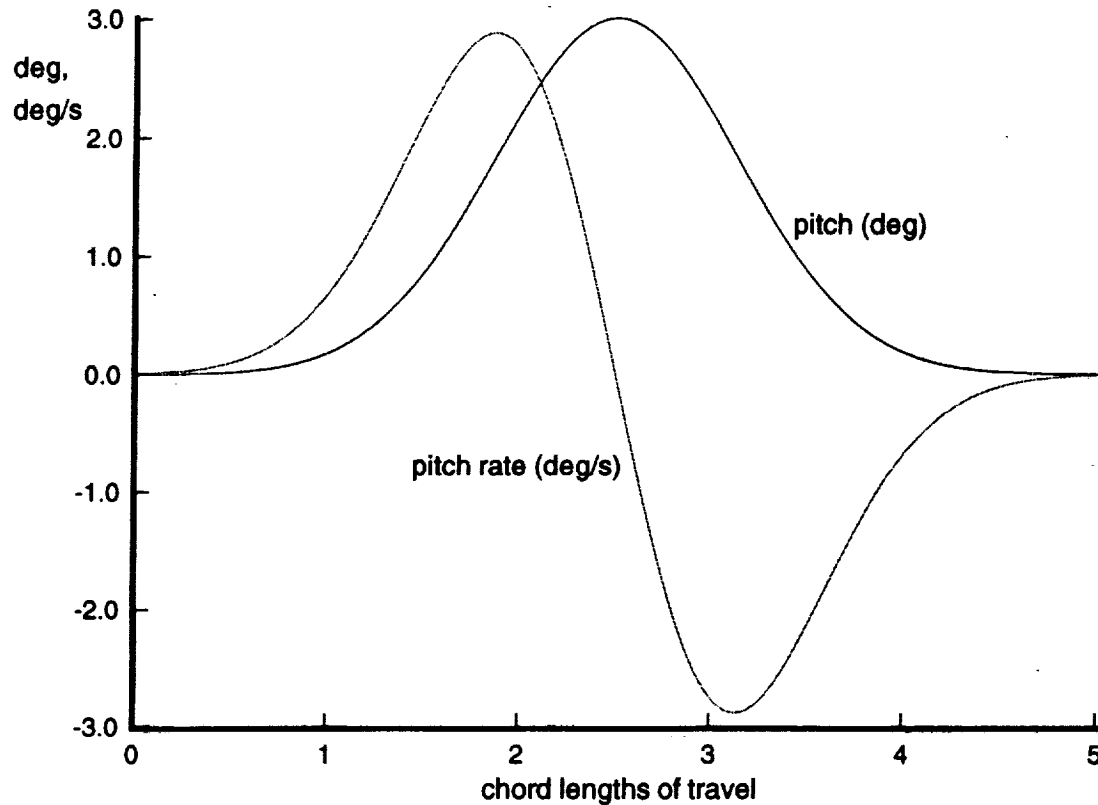
# LINEAR (FLAT PLATE) UNIT PULSE RESPONSE IN LIFT DUE TO SECOND COMPONENT OF PITCHING MOTION



## Linear (flat plate) Unit Pulse Response in Lift Due to Second Component of Pitching Motion

This is the unit pulse response in lift due to the second component of the pitching motion, or the downwash. Again, this response is stable, or square integrable. In order to validate that these responses are indeed unit pulse responses, an arbitrary pitching motion was generated.

## LOW FREQUENCY PITCHING MOTION

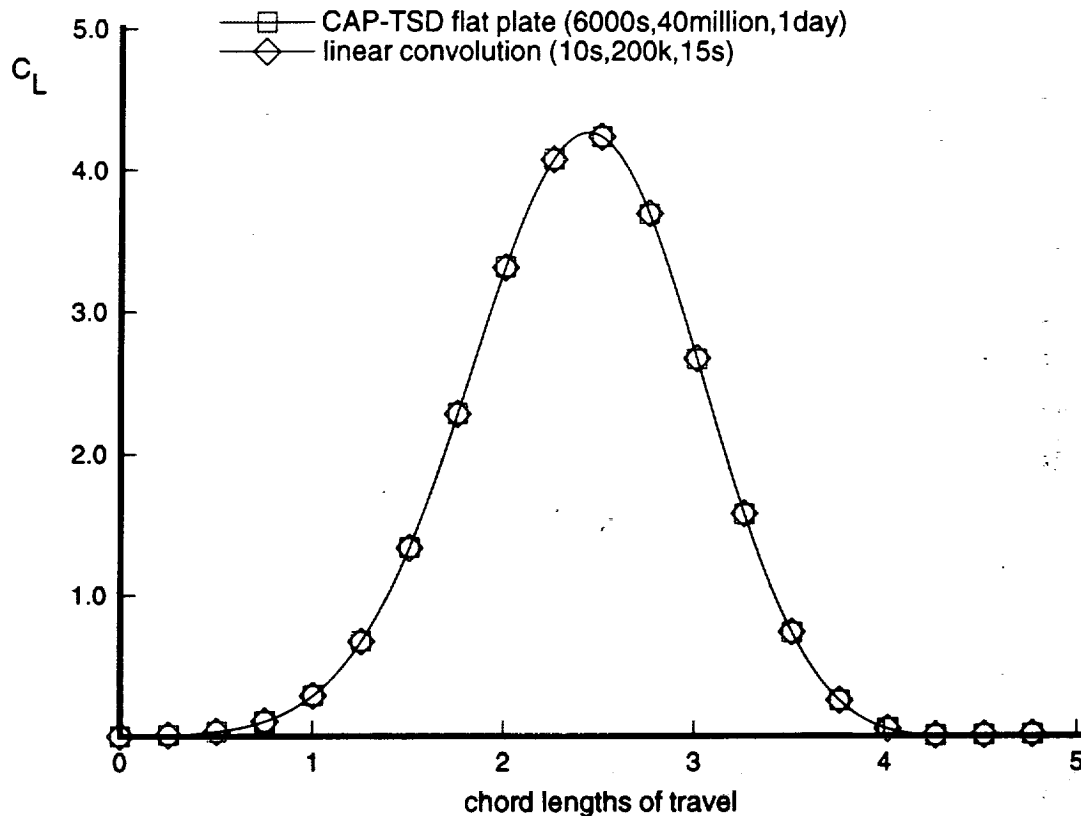


### Low Frequency Pitching Motion

This is the pitching motion that was generated. It consists of a positive pitch up to 3 degrees and then back down to 0 degrees. The corresponding rate-of-change of motion is also presented. This motion was then processed through the CAP-TSD code to obtain the CAP-TSD flat plate solution. The pitching motion was convoluted with the first unit pulse response presented and the rate-of-change of pitching motion was convoluted with the second unit pulse response presented. These two convolutions were then added to obtain the total linear convolution response.



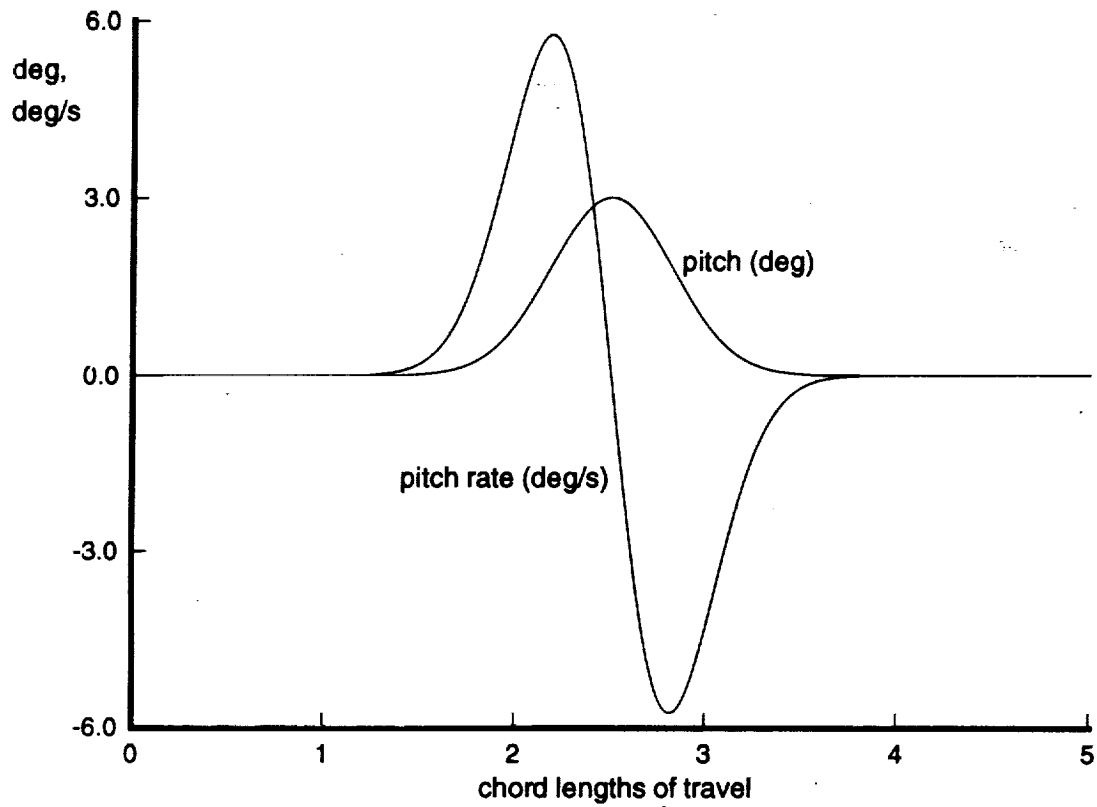
## LIFT DUE TO LOW FREQUENCY PITCHING MOTION



### Lift Due to Low Frequency Pitching Motion

This is a comparison of the CAP-TSD flat plate solution and the linear convolution solution for the low frequency pitching motion. As can be seen, the comparison is excellent yielding identical responses to plotting accuracy. It is important to note the savings in cost that was obtained by using the convolution procedure. The CAP-TSD solution cost 6000 cpu seconds, required 40 million words of memory, and was available the next day. The convolution solution cost 10 cpu seconds, required 200 thousand words of memory, and was available in 15 seconds. For linear results this is, of course, of minimal importance since linear problems are readily solved by more efficient means than a complex CFD code. The implication, however, is that similar cost savings may be achieved for nonlinear solutions. It should also be mentioned that the cost of computing the unit pulse responses should be added to the total cost of the convolution solution, but that cost was only 2400 cpu seconds. The real benefit to be obtained from the Volterra, or convolution approach, however, is that once the unit pulse responses (or kernels) are available, the same kernels can be used to predict the response to other inputs.

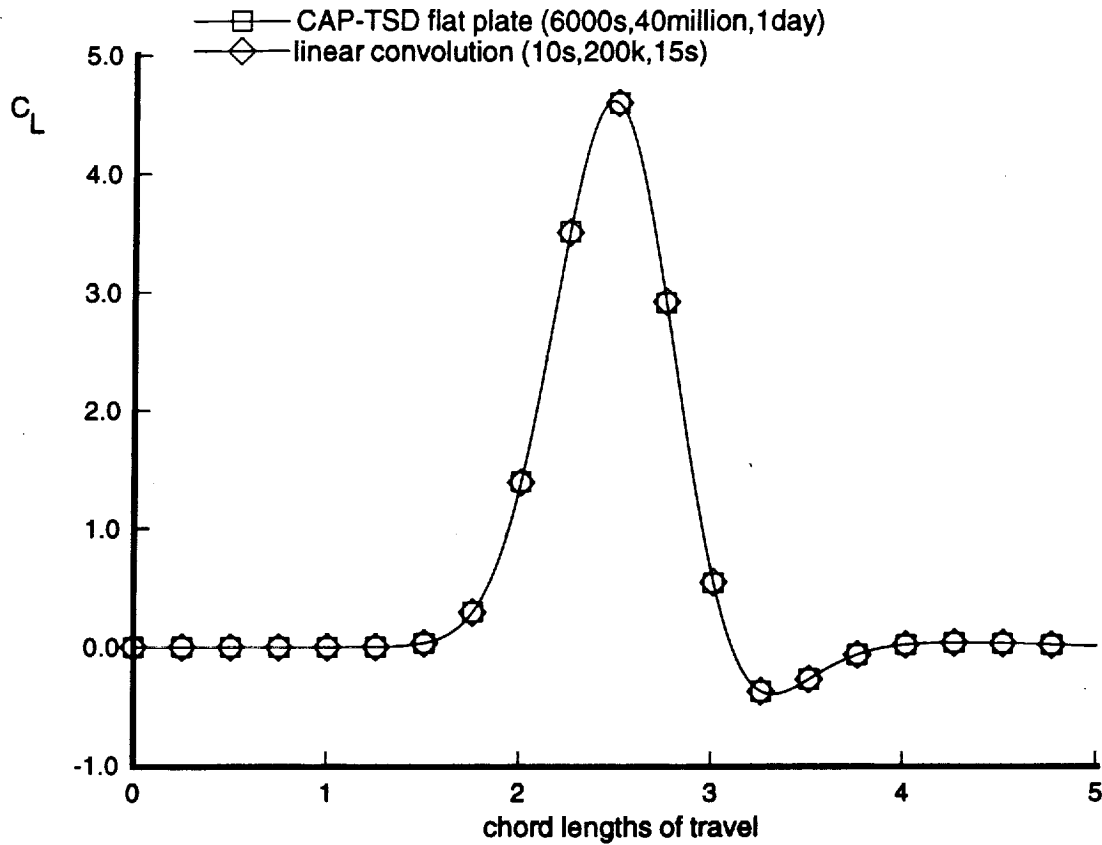
## HIGH FREQUENCY PITCHING MOTION



### High Frequency Pitching Motion

For example, if the input is now a high frequency input such as shown in the chart, convolution of this input with the corresponding unit pulse responses yields....

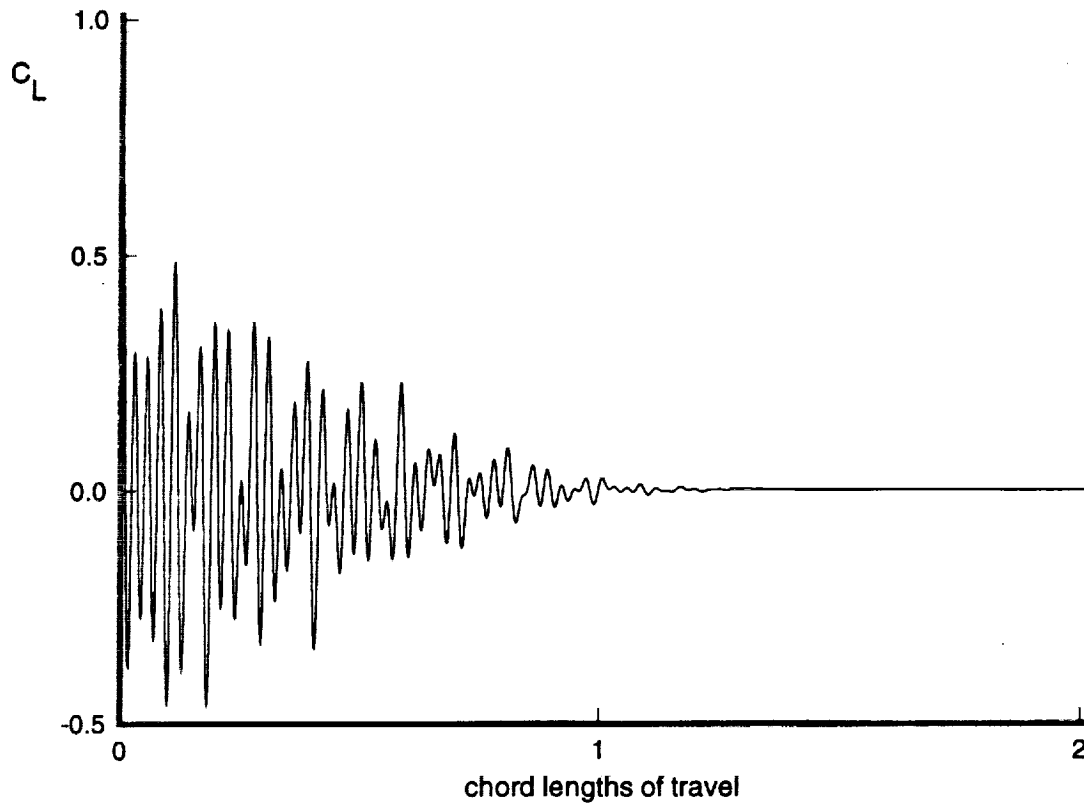
## LIFT DUE TO HIGH FREQUENCY PITCHING MOTION



### Lift Due to High Frequency Pitching Motion

... this result. Again, the comparison between the CAP-TSD flat plate solution and the linear convolution is excellent. Although the cost of the CAP-TSD solution is once again the same as that of the previous low-frequency result, the cost of the convolution is as shown on the chart. The cost of the kernel computation was paid initially and is not paid again.

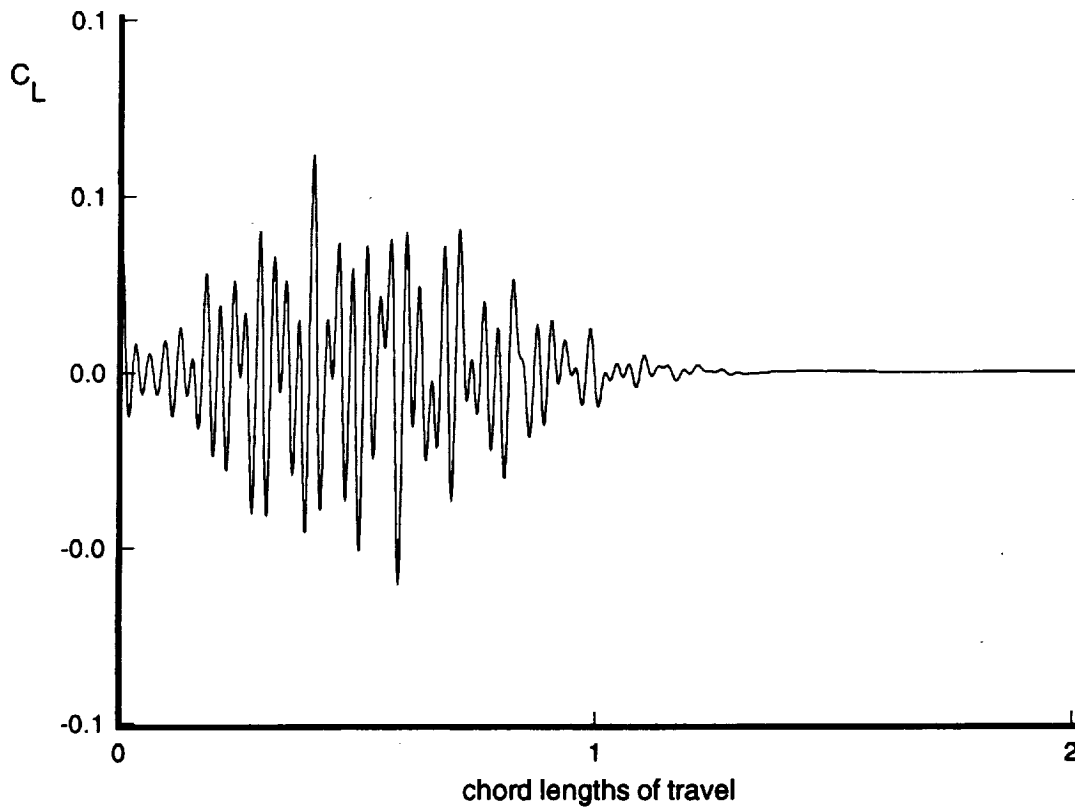
## FIRST-ORDER UNIT PULSE RESPONSE IN LIFT DUE TO FIRST COMPONENT OF PITCHING MOTION



### First-Order Unit Pulse Response Due to First Component of Pitching Motion

Investigation of the nonlinear responses begins with the computation of the first-order kernel. It is important to realize that the first-order kernel is the linear portion of the nonlinear response which is not, in general, equivalent to the purely linear response. Shown in this chart is the first-order unit pulse response due to the first component of the pitching motion. Although this response has a similar characteristic to the purely linear unit pulse response shown previously, when plotted together noticeable differences are noticeable.

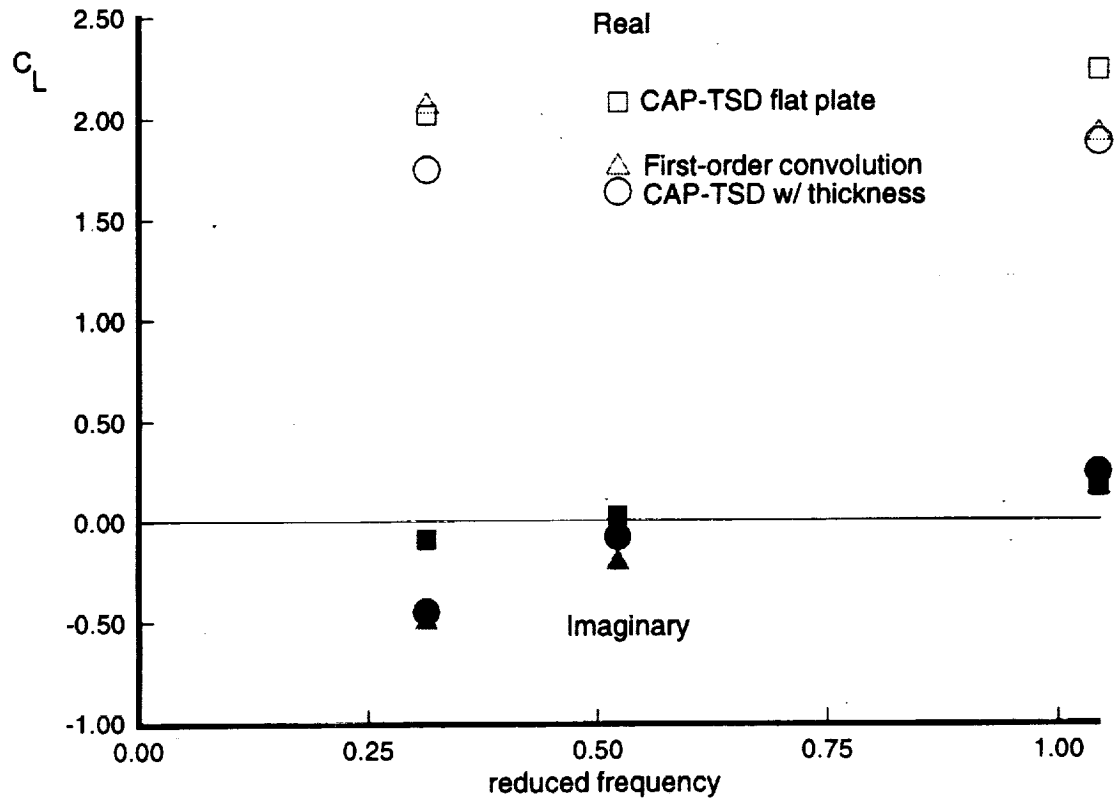
# FIRST-ORDER UNIT PULSE RESPONSE IN LIFT DUE TO SECOND COMPONENT OF PITCHING MOTION



## First-Order Unit Pulse Response Due to Second Component of Pitching Motion

This is the first-order unit pulse response due to the second component of the pitching motion. Again, a similar characteristic to the linear, or flat plate, response but it is different.

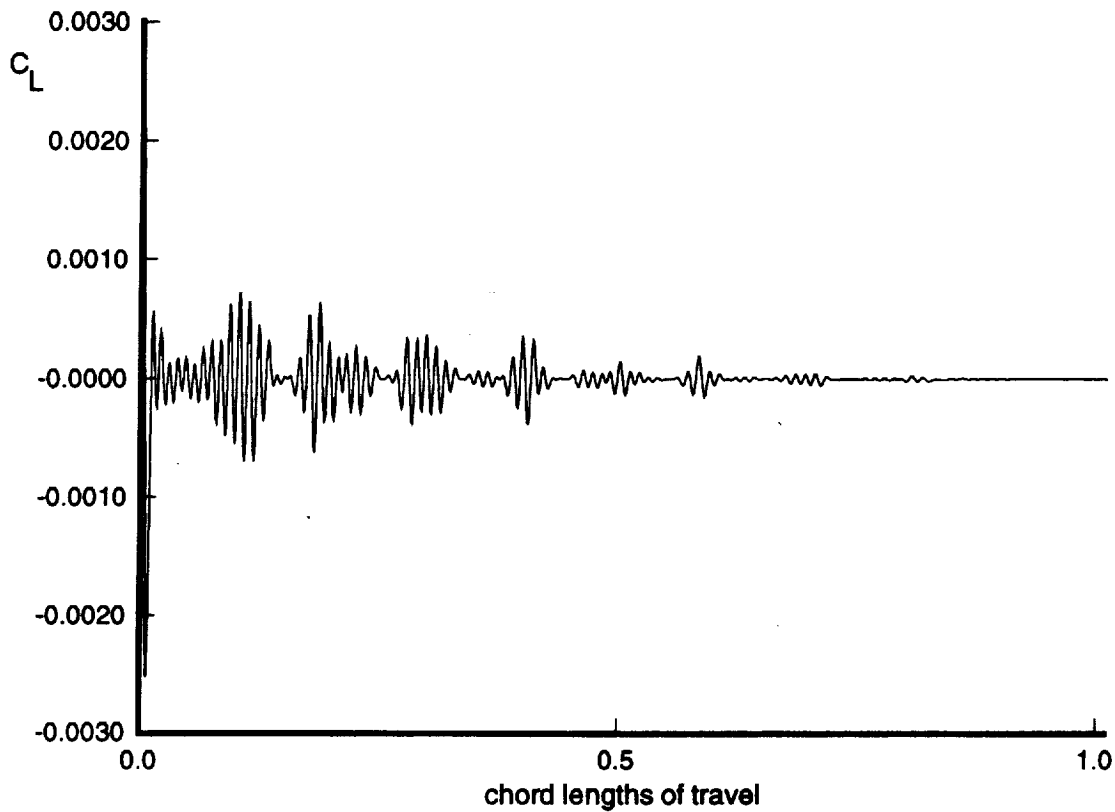
## LIFT DUE TO FORCED HARMONIC PITCHING MOTIONS



### Lift Responses Due to Forced Harmonic Pitching Motions

The first-order kernel was evaluated using forced harmonic pitching motions at three reduced frequencies of motion and compared with CAP-TSD flat plate and CAP-TSD with thickness results. The data indicates that the first-order kernel predicts the CAP-TSD nonlinear (with thickness) result at the high frequency. This comparison is degraded as reduced frequency is lowered which is to be expected since the transonic nonlinearities become more dominant as frequency is reduced. This indicates a need for the second-order kernel responses. Of interest is once again the cost savings. The three first-order responses were generated in about half an hour whereas the CAP-TSD results lasted several days and cost significantly more in CPU and memory.

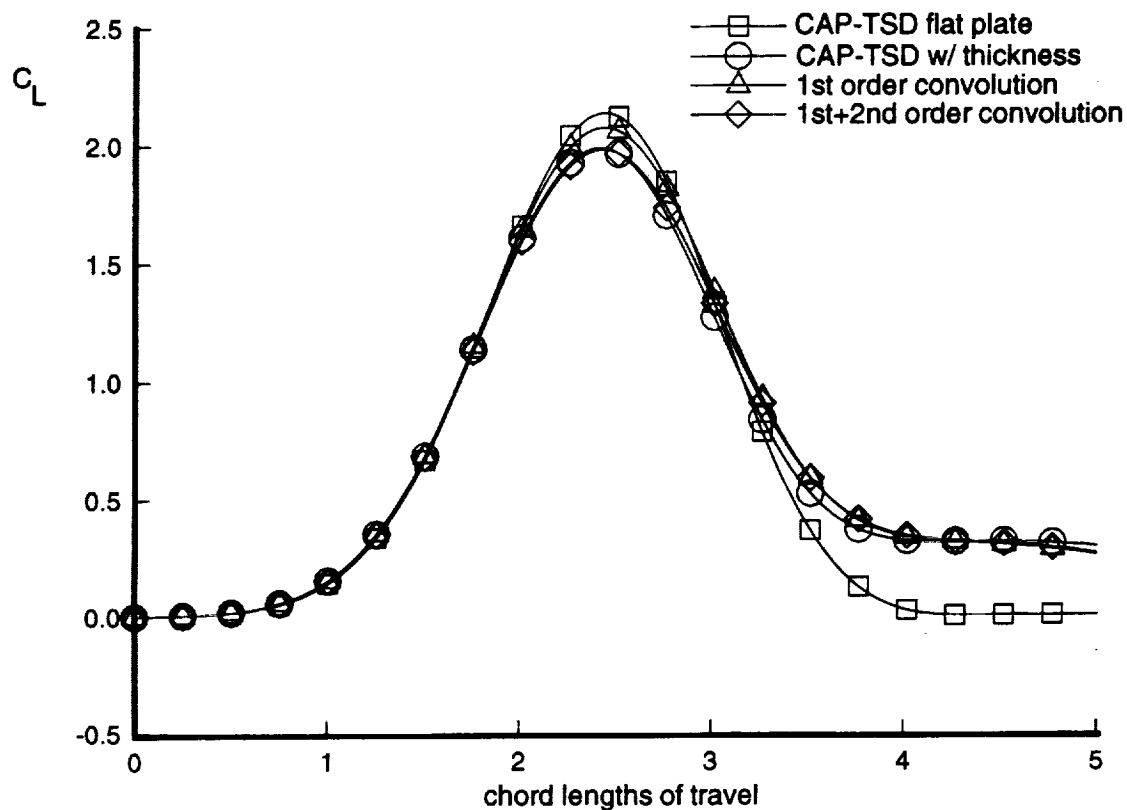
## SECOND-ORDER (NONLINEAR) UNIT PULSE RESPONSE IN LIFT DUE TO FIRST COMPONENT OF PITCHING MOTION



### Second-Order Nonlinear Unit Pulse Response Due to First Component of Pitching Motion

Shown here is the first term of the second-order kernel, or unit pulse response, due to the first component of the pitching motion. Note the noticeably different characteristic of this response as compared to the two previously shown responses. A total of four terms of the second-order kernel were computed for the present analysis.

## LIFT DUE TO LOW FREQUENCY PITCHING MOTION



### Lift Due to Low Frequency Pitching Motion

Shown here is a comparison of the responses obtained for the low frequency pitching motion: the CAP-TSD flat plate solution, the CAP-TSD with thickness solution, the first-order convolution, and the summation of the first- and second-order convolutions. It is obvious that the purely linear response, the CAP-TSD flat plate response, is quite different from the CAP-TSD with thickness response. The first-order solution, however, although it overshoots the CAP-TSD nonlinear solution (with thickness), is an improvement over the linear response. This is most notable in the latter part of the responses. The addition of the second-order terms provides the necessary difference to the first-order solution to accurately predict the peak of the CAP-TSD nonlinear solution, with very slight discrepancies near the latter part of the responses.



## ADVANTAGES OF METHODOLOGY

- CFD code used initially to define kernels
- Once kernels are defined, CFD code NOT USED AGAIN
- Linear and nonlinear responses computed using simple convolution subroutine (negligible cost)
- Kernels can be used to generate linear and nonlinear state-space matrices that define the unsteady response of the aerodynamic system

### Advantages of Methodology

The advantages of the methodology are as follows. First, the CFD code is used initially to define the necessary kernels. Once the kernels are defined, the CFD code need not be used again. This is where the potential for significant cost savings becomes obvious. Second, once the kernels are defined, linear and nonlinear responses can be computed using simple convolution routines at a negligible computational cost. Finally, from the kernels, linear and nonlinear state-space matrices can be generated that define the unsteady response of the aerodynamic system.

## **CONTINUED DEVELOPMENT (PLANS)**

- Second-order kernel definition, application, limitations
- DAVINCI (Definition of Aerodynamic Volterra Integrals for Nonlinear Control Interactions) Team formed with Aeroservoelasticity Branch (Mukhopadhyay, Wieseman)
- System realization, bilinear equations
- Apply methodology to Euler/Navier-Stokes code(s)

### **Continued Development (Plans)**

Current efforts are aimed at additional second-order kernel definition and validation, applications, and limitations. The DAVINCI (Definition of Aerodynamic Volterra Integrals for Nonlinear Control Interactions) Team has been formed with Dr. Vivek Mukhopadhyay and Carol Wieseman of the same branch. Additional work is being performed in understanding the system realization issues for bilinear systems and bilinear equations. It is also planned to apply kernel identification techniques to higher-level fluid dynamics equations such as the Euler and Navier-Stokes equations.

## CONCLUDING REMARKS

- Linear and nonlinear discrete aerodynamic unit pulse response functions (kernels) defined for arbitrary frequencies
- Linear (flat-plate) results : excellent
- Nonlinear results
  - First-order term provides "linearized" result
  - Second-order term provides nonlinear effect
  - Additional validation/development underway
- Cost savings (CPU, memory and turnaround time)

### Concluding Remarks

In conclusion, linear and nonlinear discrete aerodynamic unit pulse response functions (kernels) were defined for arbitrary frequencies. The fact that these functions exist is of significance as it represents an approach different to the indicial method. Linear, or flat plate, results were excellent in comparison with the CAP-TSD flat plate generated results. The linear results validate the use of unit pulse responses for aerodynamic systems. The nonlinear results are very encouraging in that, for the responses investigated, the first-order term provides the "linearized" result and the second-order term captures the nonlinear effect. There is, of course, additional validations and development work that needs to be performed to fully understand the effectiveness and the limitations of the methodology. As was shown, the cost savings is significant for the cases shown, which would make CFD codes practical for preliminary analysis and design.

